## M20580 L.A. and D.E. Tutorial Worksheet 13

1. Give the form of a particular solution $y_{p}$ to each of the following nonhomogeneous linear differential equations. Do not find the values of the constants.
(a) $y^{\prime \prime}+4 y^{\prime}+4 y=x e^{7 x}$
(b) $y^{\prime \prime}+9 y=\sin (2 x)$
(c) $y^{\prime \prime}+9 y=\sin (3 x)$
(d) $y^{\prime \prime}-6 y^{\prime}+9 y=x^{2}+e^{x} \cos (4 x)$,
(e) $y^{\prime \prime}-6 y^{\prime}+9 y=3 e^{3 x}-x e^{3 x}$
2. Consider the differential equation

$$
y^{\prime \prime}+4 y^{\prime}+4 y=x e^{-2 x}
$$

Using the method of undetermined coefficients,
(a) Find a particular solution $y_{p}$
(b) Find the general solution.
3. Using variation of parameters, find a particular solution for

$$
y^{\prime \prime}+4 y=\sec (2 t)
$$

4. The differential equation

$$
\left(x^{2}-2 x\right) y^{\prime \prime}+\left(2-x^{2}\right) y^{\prime}+(2 x-2) y=0
$$

has solutions $y_{1}(x)=e^{x}$ and $y_{2}(x)=x^{2}$.
(a) Find the associated Green's function, $G(x, t)$.
(b) Use the Green's function to find a particular solution of the differential equation

$$
\left(x^{2}-2 x\right) y^{\prime \prime}+\left(2-x^{2}\right) y^{\prime}+(2 x-2) y=-\left(2 x-x^{2}\right)^{2} e^{x}
$$

satisfying $y_{p}(0)=0$ and $y_{p}^{\prime}(0)=0$
5. Solve the initial-value problem: $y^{\prime \prime}+y=-3 \sin (x) \cos (x), y(0)=3, y^{\prime}(0)=4$

