Name: Megan O'Donnell Date: 05/06/2021

M20580 L.A. and D.E. Tutorial
Quiz 12

d 1. Given that $y_1 = e^{-2t}$ and $y_2 = e^{2t}$ form a fundamental set of solutions to $y'' - 4y = 0$, we know $y'' - 4y = e^{2t}t^{-4}$ has a particular solution $y_p = u_1(t)e^{-2t} + u_2(t)e^{2t}$ given by a variation of parameters. Find a valid $u_2(t)$.

(a) $u_2(t) = -2t^{-3}$ (b) $u_2(t) = -\frac{t^{-3}}{12}$ (c) $u_2(t) = \frac{t^{-3}}{6}$ (d) $u_2(t) = \frac{t^{-2}}{12}$
(e) $u_2(t) = -\frac{t^{-2}}{6}$

d 2. Which of the following is the Green's function $G(x,t)$ for the linear operator

$$L(y) = \frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y?$$

(a) $\frac{e^{-t}e^{x} - e^{-3x}e^{3t}}{4}$ (b) $\frac{e^{-4t}e^{4x} - e^{-2x}e^{2t}}{2}$ (c) $\frac{e^{-4t}e^{4x} - e^{-2x}e^{2t}}{4}$
(d) $\frac{e^{-3t}e^{3x} - e^{-x}e^{t}}{2}$ (e) $\frac{e^{-3t}e^{3x} - e^{-x}e^{t}}{4}$

$G(x,t) = \frac{\qquad}{W(e \quad)(t)}$

$m^2 - 2m - 3 = 0$

$(m-3)(m+1) = 0$ $m = -1, 3$

$$W = \begin{vmatrix} e^{-3t}e^{3x} & e^{-x}e^{t} \\ -3e^{3t}e^{3x} + 3e^{-3t}e^{3x} & -e^{-x}e^{t} + \cancel{e^{-x}e^{t}} \end{vmatrix}$$

$W = -(e^{-x}e^{t})(\cancel{-3e^{3t}e^{3x} + 3e^{-3t}e^{3x}})$

$W = 0$